# Query Optimization: Exercise 

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# Direct, Uniform, Distinct: Yao 

Given $m$ pages with $n$ tuples on each page, e.g. a total of $N=m \cdot n$ tuples:


- How many distinct subsets of size $k$ exist? $\binom{N}{k}$
- How many distinct subsets of size $k$ exist, where a page does not contain any of the chosen tuples? Choose $k$ from all but one page, i.e. from $N-n$ tuples: $\binom{N-n}{k}$ So the probability that a page contains none of the $k$ tuples is

$$
p:=\frac{\binom{N-n}{k}}{\binom{N}{k}}
$$

- What is the probability that a certains page contains at least one tuple? $1-p \ldots$ unless all pages have to be involved $(k>N-n)$.
- Multiplied by the number of pages, we get the number of qualifying pages, denoted $\overline{\mathcal{Y}}_{n}^{N, m}(k)$.


# Direct, Uniform, Non-Distinct: Cheung 

- Now with replacement: How many distinct multisets exist chosing $k$ from $n$ ? As many as there are distinct sets chosing $k$ from $n+k-1$ !
- Bijection between multisets and sets. From multiset to set: $f:\left(x_{1}, x_{2}, \ldots, x_{k}\right) \mapsto\left(x_{1}+0, x_{2}+1, \ldots, x_{k}+(k-1)\right)$
- Example: Choose 2 from 4
- \# sets: $\binom{4}{2}$
- \# multisets: $\binom{4+2-1}{2}$

- Like Yao, but not necessarily distinct
- Same formula as Yao, but:
- No special case for $k>N-n$
- We substitue $N$ by $N+k-1$ to compute $\tilde{p}$


## Sequential, Uniform, Distinct

- Estimate the distribution of distance between two qualifying tuples
- Bitvector $B, b$ bits are set to 1
- First, the distribution of the number of $j$ zeros
- before first 1
- between two consecutive 1s
- after last 1
- Bitvectors having a 1 at position i followed by j zeros: $\binom{B-j-2}{b-2}$
- $B-j-1$ positions for $i$
- every bitvector has $b-1$ sequences of a form $10 \ldots 01$
- $\mathcal{B}_{b}^{B}(j)=\frac{(B-j-1)\binom{B-j-2}{b-2}}{(b-1)\binom{B}{b}}=\frac{\binom{B-j-1}{b-1}}{\binom{B}{b}}$
- now, the expected number of $0 \mathrm{~s}: \frac{B-b}{b+1}$
- then, the expected total number of bits between first and last 1: $B-\frac{B-b}{b+1}=\frac{B b+b}{b+1}$

Histograms

A histogram $H_{A}: B \rightarrow \mathbb{N}$ over a relation $R$ partitions the domain of the aggregated attribute $A$ into disjoint buckets $B$, such that

$$
H_{A}(b)=|\{r \mid r \in R \wedge R . A \in b\}|
$$

and thus $\sum_{b \in B} H_{A}(b)=|R|$.

A rough histogram might look like this:


Given a histogram, we can approximate selectivities as follows:

$$
\begin{array}{ll}
A=c & \frac{\sum_{b \in B: c \in b} H_{A}(b)}{\sum_{b \in B} H_{A}(b)} \\
A>c \quad & \frac{\sum_{b \in B: c \in b} \frac{\max (b)-c}{\max (b)-\min (b)} H_{A}(b)+\sum_{b \in B: \min (b)>c} H_{A}(b)}{\sum_{b \in B} H_{A}(b)} \\
A_{1}=A_{2} & \frac{\sum_{b_{1} \in B_{1}, b_{2} \in B_{2}, b^{\prime}=b_{1} \cap b_{2}: b^{\prime} \neq \emptyset} \frac{\max \left(b^{\prime}\right)-\min \left(b^{\prime}\right)}{\max \left(b_{1}\right)-\min \left(b_{1}\right)} H_{A_{1}}\left(b_{1}\right) \frac{\max \left(b^{\prime}\right)-\min \left(b^{\prime}\right)}{\max \left(b_{2}\right)-\min \left(b_{2}\right)} H_{A_{2}}\left(b_{2}\right)}{\sum_{b_{1} \in B_{1}} H_{A_{1}}\left(b_{1}\right) \sum_{b_{2} \in B_{2}} H_{A_{2}}\left(b_{2}\right)}
\end{array}
$$

Given the following histogram of an integer attribute R.a:

| bucket | $[0,20)$ | $[20,40)$ | $[40,60)$ | $[60,80)$ | $[80,100)$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| count | 1 | 3 | 4 | 2 | 0 |

Estimate the number of elements for which $R . a>=55$ holds true.

- Slides and exercises: db.in.tum.de/teaching/ws1718/queryopt
- Send any questions, comments, solutions to exercises etc. to radke@in.tum.de
- No more exercises.

